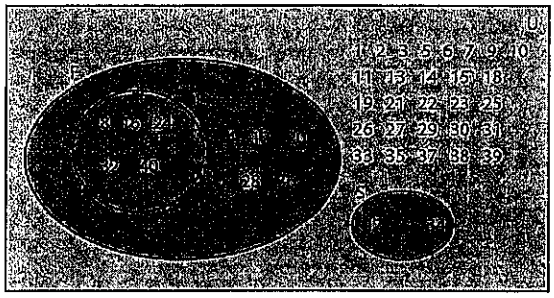
Chapter 1: Set Theory

Lesson 1.1: Types of Sets and Set Notation, page 14

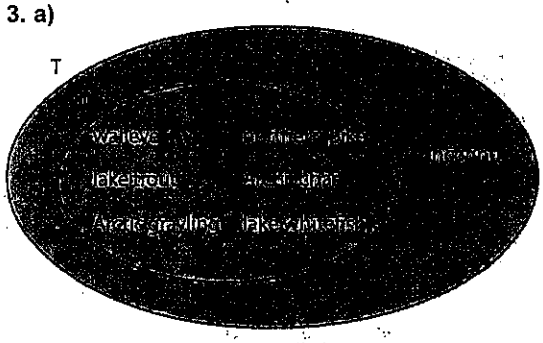
1. a) e.g., Yes, those explanations make sense.
b) The universal set is set C.
C = {produce}
O = {orange produce} = {oranges, carrots}
Y = {yellow produce} = {bananas, pineapple, corn}
G = {green produce} = {apples, pears, peas, beans}
B = {brown produce} = {potatoes, pears}
c) e.g., $S \subset F$ because all fruits you can eat without peeling are also fruits. $S \subset C$ because all fruits you can eat without peeling are also produce.
d) Sets S and V are disjoint sets, as are F and V.
e) Yes. e. g., C = {F and V}, $F' = V$.
f)
$$n(V) = n(C) - n(F)$$
$$n(V) = 10 - 5$$
$$n(V) = 5$$
g) oranges, pineapple, bananas, peas, corn, carrots, beans, potatoes

2. a)



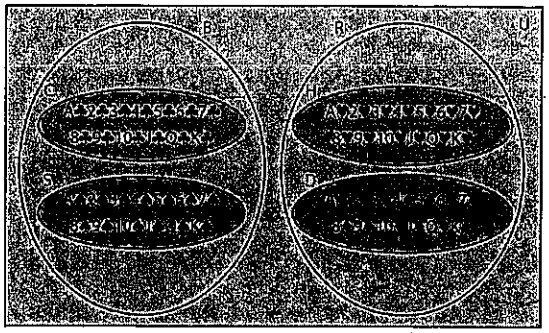
b) Sets E and S are disjoint sets, as are sets F and S.
c) i) True. e.g., Multiples of 8 are also multiples of 4.
ii) False. e.g., Not all multiples of 4 are multiples of 8.
iii) True. e.g., All multiples of 4 are multiples of 4.
iv) False. e.g., F' = {all numbers from 1 to 40 that are not multiples of 4}
v) True. e.g., The universal set includes natural numbers from 1 to 40.

3. a)



b) e.g., $N \subset T$ means that all the fish found in Nunavut are also found in the Northwest Territories. $T \not\subset N$ means that not all the fish found in the Northwest Territories are found in Nunavut.

4. a)



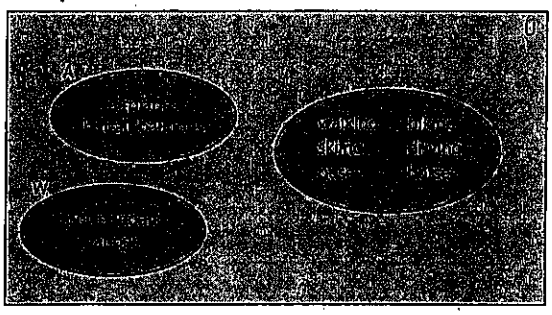
b) Subsets of set B: $C \subset B$ and $S \subset B$
c) Subsets of set R: $H \subset R$ and $D \subset R$
d) Yes, the sets S and C are disjoint. e.g., A card cannot be both a spade and a club.
e) Yes, the events in sets H and D are mutually exclusive. e.g., You cannot draw a card that is a heart and a diamond at the same time.
f) Yes, that statement is correct. e.g., Because these sets are disjoint, they contain no common elements. Therefore, when the numbers of elements in each set are added, no element will be counted twice.
$n(S \text{ or } D) = n(S) + n(D)$
$n(S \text{ or } D) = 13 + 13$
$n(S \text{ or } D) = 26$

5. a) e.g., C = {all clothes}, S = {summer clothes}, W = {winter clothes}, H = {summer headgear}
b) e.g., In set C, but not in set S or set W, because they would be worn year round.
c) No, set S' is not equal to set W. Set S' includes the jacket, but W does not.
d) Sets S and W are disjoint sets. Sets H and W are disjoint sets.
e) e.g., C = {clothes},
H = {headgear} = {cap, sunglasses, toque},
B = {clothing for body} = {shirt, shorts, coat, jacket}, F = {footwear} = {sandals, insulated boots}

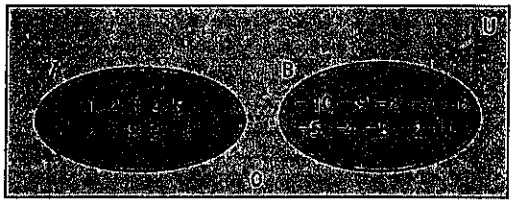
6.
$$n(X') = n(U) - n(X)$$
$$n(X') = 100\,000 - 12$$
$$n(X') = 99\,988$$
7. Not possible; e.g., there may be some elements that are in both X and Y.

8.
$$n(U) = n(X) + n(X')$$
$$n(U) = 34 + 42$$
$$n(U) = 76$$
9. a) S = {A, E, F, H, I, K, L, M, N, T, V, W, X, Y, Z}
C = {C, O, S}
b) False. e.g., B is not in S or C.

10. Let U represent the universal set. Let L represent the set of land transportation. Let W represent the set of water transportation. Let A represent the set of air transportation.



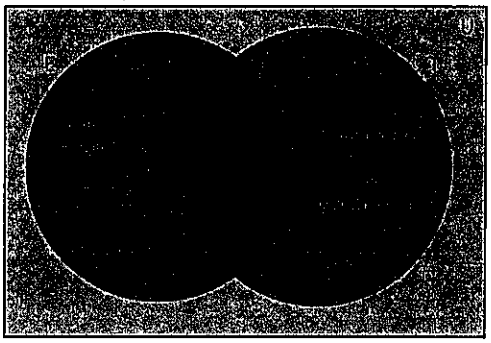
11. a)



b) Sets A and B are disjoint sets.
c) i) False. e.g., 1 is not in B.
ii) False. e.g., -1 is not in A.
iii) False. e.g., 0 is in A' but not in B.
iv) True. e.g., $n(A) = 10$, $n(B) = 10$.
v) True. e.g., No integer from -20 to -15 is in U.

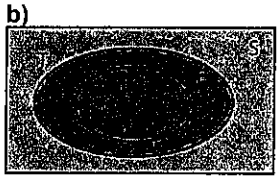
12. a) $S = \{4, 6, 9, 10, 14, 15, 21, 22, 25, 26, 33, 34, 35, 38, 39, 46, 49\}$
$W = \{1, 2, 3, 5, 7, 8, 11, 12, 13, 16, 17, 18, 19, 20, 23, 24, 27, 28, 29, 30, 31, 36, 37, 40, 41, 42, 43, 44, 45, 47, 48, 50\}$
b) e.g., E = {even semiprime numbers}
$E = \{4, 6, 10, 14, 22, 26, 34, 38, 46\}$
c) $n(W) = n(U) - n(S)$
$n(W) = 50 - 17$
$n(W) = 33$
d) No, it is not possible to determine $n(A)$. e.g., There is an infinite number of prime numbers, so there is an infinite number of semiprime numbers.

13. e.g., Let U represent the universal set. Let E represent the set of entertainment items. Let T represent technology items.

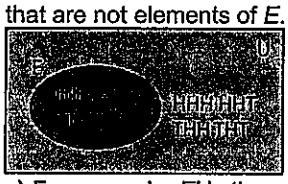


14. Agree; e.g., $A \subset B$ means that set A is a part of set B, and it could be that set A and set B are equal. If $A \subset B$, then set A will have the same number or fewer elements than set B. With numbers, $x \leq y$ means that x is less than or equal to y. Or, or if $A \subset B$, then $n(A) \leq n(B)$. The number of elements in a subset must be equal to or less than the number of elements in the set.

15. a) $S = \{x \mid -1000 \leq x \leq 1000, x \in I\}$
$T = \{t \mid t = 25x, -40 \leq x \leq 40, x \in I\}$
$F = \{f \mid f = 50x, -20 \leq x \leq 20, x \in I\}$
$F \subset T \subset S$
b)

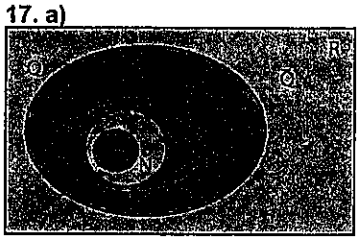


16. a) U = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}
b) E = {HTH, HTT, TTH, TTT}
c) $n(U) = 8$, $n(E) = 4$
d) Yes, e.g., because each element of E is also an element of U, and there are some elements of U that are not elements of E.



e) For example, E' is the set of elements of U where the second coin turns up heads.
$n(E') = n(U) - n(E)$
$n(E') = 8 - 4$
$n(E') = 4$
E' = {HHH, HHT, THH, THT} and $n(E') = 4$
f) Yes. e.g., A coin cannot show both heads and tails at the same time.

17. a)



b) e.g., N' is the set of all non-natural numbers. W' is the set of all non-whole numbers. I' is the set of non-integer numbers. Q' is the set of numbers that cannot be described as a ratio of two integers. $\overline{Q}$ is the set of numbers that can be described as a ratio of two integers.

Set	Complement
N	$N' = \{x \mid x \in R, x \notin N\}$
I	$I' = \{x \mid x \in R, x \notin I\}$
Q	$\overline{Q}$
$\overline{Q}$	Q

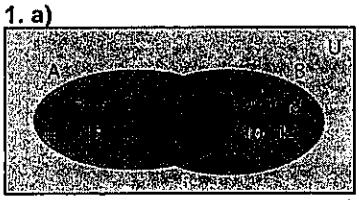
c) Sets N and $\overline{Q}$ are disjoint sets. Sets W and $\overline{Q}$ are disjoint sets. Sets I and $\overline{Q}$ are disjoint sets. *Sets* Q and $\overline{Q}$ are disjoint sets.
d) Yes. e.g., Q' is the set of numbers that cannot be described as a ratio of two integers, which is the set of irrational numbers.
e) W, I, Q, R
f) No. e.g., The area of a region in a Venn diagram is not related to the number of elements in the set.

18. a) $S = \{1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289\}$
$n(S) = 17$
$E = \{4, 16, 36, 64, 100, 144, 196, 256\}$
$n(E) = 8$
b) $n(S) = 17$, $n(E) = 8$
$n(O) = n(S) - n(E)$
$n(O) = 17 - 8$
$n(O) = 9$
c) $n(U) = 300$, $n(S) = 17$
$n(S') = n(U) - n(S)$
$n(S') = 300 - 17$
$n(S') = 283$

19. a) e.g., $A \subset B$ if all elements of A are also in B. For example, all weekdays are also days of the week, so weekdays is a subset of days of the week.
b) e.g., A' consists of all the elements in the universal set but not in A. For example, all days of the week that are not weekdays are weekend days. So weekend days is the complement of weekdays.
20. e.g., Disagree; since both the subsets are empty, they both contain the same elements and are therefore the same subset.

Lesson 1.2: Exploring Relationships between Sets, page 20

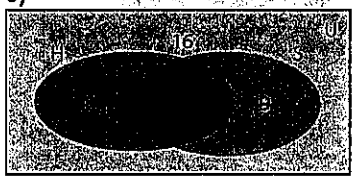
1. a)



b) i) $n(A) = 5$
ii) $n(A \text{ but not } B) = n(A) - n(A \text{ and } B)$
$n(\text{A but not } B) = 5 - 2$
$n(\text{A but not } B) = 3$
iii) $n(B) = 7$
iv) $n(B \text{ but not } A) = n(B) - n(A \text{ and } B)$
$n(B \text{ but not } A) = 7 - 2$
$n(B \text{ but not } A) = 5$
v) $n(A \text{ and } B) = 2$
vi) $n(A \text{ or } B) = n(\text{A but not } B) + n(A \text{ and } B) + n(B \text{ but not } A)$
$n(A \text{ or } B) = 3 + 2 + 5$
$n(A \text{ or } B) = 10$
vii) $n(A) = 5$, therefore $n(A') = 5$

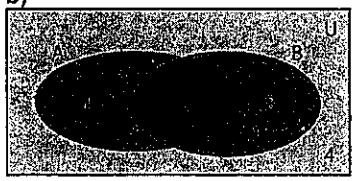
2. a) 8 students are in both the drama club and the band.
b) 11 students are in the drama club only.
6 students are in the band only.
c) Drama: $11 + 8 = 19$
Band: $8 + 6 = 14$
d) Drama club or band: $11 + 8 + 6 = 25$
e) 38 students in grade 12 – 25 in drama club or band = 13 students in neither drama club nor band

3. a) hockey or soccer: $45 - 16 = 29$
hockey and soccer: $20 + 14 = 34$
overlap: $34 - 29 = 5$
5 students like hockey and soccer.
b) only hockey: $20 - 5 = 15$
only soccer: $14 - 5 = 9$
$15 + 9 = 24$
24 students like only hockey or only soccer.
c)



4. a) ski or snowboard: $55 - 9 = 46$
ski and snowboard: $25 + 32 = 57$
Overlap: $57 - 46 = 11$
11 guests plan to ski and snowboard.
b) only ski: $25 - 11 = 14$
14 guests will only ski.
c) only snowboard: $32 - 11 = 21$
21 guests will only snowboard.

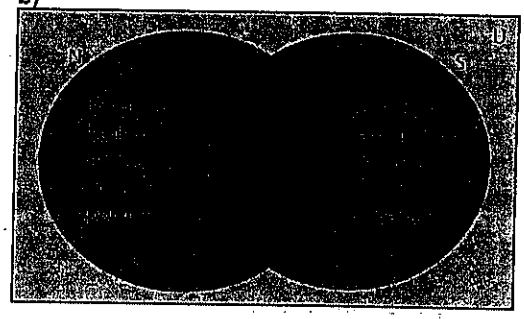
5. a) $n(U) - n(U \text{ but not } A \text{ or } B)$: $25 - 4 = 21$
$n(A) + n(B)$: $13 + 10 = 23$
$n(A \text{ and } B)$: $23 - 21 = 2$
$n(A \text{ only})$: $13 - 2 = 11$
$n(B \text{ only})$: $10 - 2 = 8$
b)



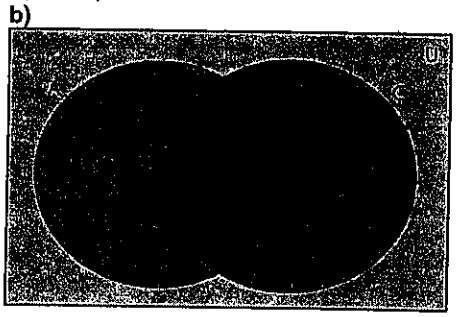
Lesson 1.3: Intersection and Union of Two Sets, page 32

1. a) $A = \{-10, -8, -6, -4, -2, 0, 2, 4, 6, 8, 10\}$
$B = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
$A \cup B = \{-10, -8, -6, -4, -2, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
b) $n(A \cup B) = 16$
c) $A \cap B = \{0, 2, 4, 6, 8, 10\}$
d) $n(A \cap B) = 6$

2. a) Let A represent the universal set. Let N represent the set of tundra animals. Let S represent the set of southern animals.
$N = \{$arctic fox, caribou, ermine, grizzly bear, muskox, polar bear$\}$
$S = \{$bald eagle, Canadian lynx, grey wolf, grizzly bear, long-eared owl, wolverine$\}$
$N \cup S = \{$Arctic fox, caribou, ermine, muskox, polar bear, grizzly bear, bald eagle, Canadian lynx, grey wolf, long-eared owl, wolverine$\}$
$T \cap S = \{$grizzly bear$\}$
b)

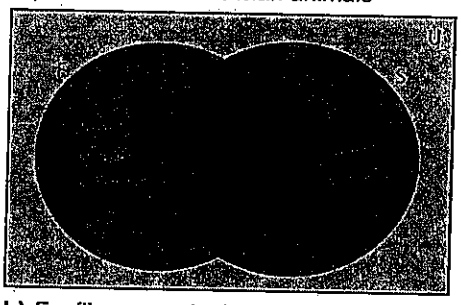


3. a) $A \cup C = \{-10, -8, -6, -4, -2, 0, 2, 4, 6, 8, 10, 12, 14, 16\}$
$n(A \cup C) = 14$
$A \cap C = \{2, 4, 6, 8, 10\}$
$n(A \cap C) = 5$
b)



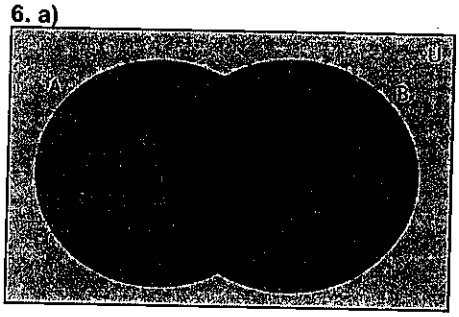
4. a) $T \cup C = \{$half-ton trucks, quarter-ton trucks, vans, SUVs, crossovers, 4-door sedans, 2-door coupes, sports cars, hybrids$\}$
b) $n(T \cup C) = 9$
c) $T \cap C = \{$crossovers$\}$

5. a) Let U represent the universal set. Let F represent the set of African animals. Let S represent the set of Asian animals



b) $F = \{$lion, camel, giraffe, hippo, elephant$\}$
$S = \{$elephant, tiger, takin, camel$\}$
$F \cup S = \{$lion, giraffe, hippo, camel, elephant, tiger, takin$\}$
$F \cap S = \{$camel, elephant$\}$

6. a)



b) $A \cup B = \{-12, -9, -6, -4, -3, -2, 0, 2, 3, 4, 6, 8, 9, 10, 12, 15\}$
$n(A \cup B) = 16$
$A \cap B = \{-6, 0, 6, 12\}$
$n(A \cap B) = 4$

7. Let U represent the universal set. Let H represent the set of people who liked Sherlock Holmes. Let P represent the set of people who liked Hercule Poirot.
$n(H \cup P) = n(U) - n((H \cup P)')$
$n(H \cup P) = 25 - 4$
$n(H \cup P) = 21$
$n(H \cap P) = n(H) + n(P) - n(H \cup P)$
$n(H \cap P) = 16 + 11 - 21$
$n(H \cap P) = 6$

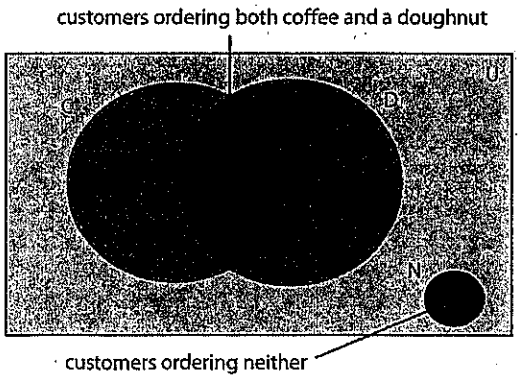
6 people like both detectives.
$n(H \text{ only}) = n(H) - n(H \cup P)$
$n(H \text{ only}) = 16 - 6$
$n(H \text{ only}) = 10$
10 people liked Sherlock Holmes only.
$n(P \text{ only}) = n(P) - n(H \cup P)$
$n(P \text{ only}) = 11 - 6$
$n(P \text{ only}) = 5$
5 people liked Hercule Poirot only.

8. Let U represent the universal set. Let V represent the set of people who liked vanilla ice cream. Let C represent the set of people who liked chocolate ice cream.
$n(C \cup V) = n(U) - n((C \cup V)')$
$n(C \cup V) = 80 - 9$
$n(C \cup V) = 71$
$n(C \text{ only}) = n(C \cup V) - n(V \text{ only}) - n(C \cap V)$
$n(C \text{ only}) = 71 - 11 - 20$
$n(C \text{ only}) = 40$
40 people like chocolate ice cream only.

9. Let U represent the universal set. Let K represent the set of people who like to ski. Let W represent the set of people who like to swim.
$n(K \cup W) = n(U) - n((K \cup W)')$
$n(K \cup W) = 26 - 5$
$n(K \cup W) = 21$
$n(K \cap W) = n(K) + n(W) - n(K \cup W)$
$n(K \cap W) = 19 + 14 - 21$
$n(K \cap W) = 12$
12 people like to ski and swim.

10. e.g., She could draw a Venn diagram showing the set of multiples of 2 and the set of multiples of 3. The intersection of the sets would be the multiples of 6.

11. a) U = {all customers surveyed}
C = {customers ordering coffee}
D = {customers ordering donuts}
N = {customers ordering neither coffee nor doughnuts}
b) For the following Venn diagram:
The rectangular area labelled U represents the universal set.
The shaded area labelled D represents the set of people who ordered doughnuts.
The shaded area labelled C represents the set of people who ordered coffee.
The shaded area labelled $D \cap C$ represents the set of people who ordered coffee and doughnuts.
The unshaded area labelled N represents those people did not order coffee or doughnuts.



c) Determine $n(D \cap C)$ using the information available.
$n(U) = 100$, $n(D) = 45$, $n(C) = 65$, $n((D \cup C)') = 10$
$n(D \cup C) = n(U) - n((D \cup C)')$
$n(D \cup C) = 100 - 10$
$n(D \cup C) = 90$
Therefore,
$n(D \cap C) = n(D) + n(C) - n(D \cup C)$
$n(D \cap C) = 45 + 65 - 90$
$n(D \cap C) = 20$
There were 20 people who ordered coffee and a doughnut.

12. Let U represent the universal set. Let T represent the set of seniors who watch television. Let R represent the set of seniors who listen to the radio.
$n(R \text{ only}) = n(U) - n(T)$
$n(R \text{ only}) = 100 - 67$
$n(R \text{ only}) = 33$
33 seniors prefer to listen to the radio only.

13. Let U represent the universal set. Let C represent the set of people who attended the Calgary Stampede. Let P represent the set of people who attended the PNE.
$n(C \cup P) = n(U) - n((C \cup P)')$
$n(C \cup P) = 56 - 14$
$n(C \cup P) = 42$
$n(C \cap P) = n(C) + n(P) - n(C \cup P)$
$n(C \cap P) = 30 + 22 - 42$
$n(C \cap P) = 10$
10 people had been to both the Calgary Stampede and the PNE.

14. Of the 54 people, 31 own their home, so $54 - 31 = 23$ people rent their home. Of that 23, 9 rent their house, so $23 - 9 = 14$ rent their condominium. Of the 30 people who live in a condominium, 14 rent, so $30 - 14 = 16$ must own the condominium in which they live.

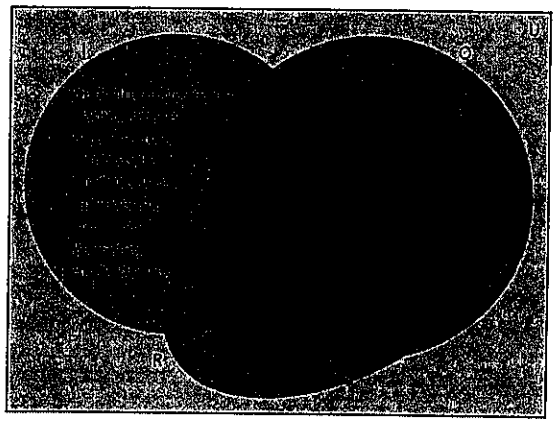
15. Let U represent the universal set. Let R represent the set of people who like reality shows. Let C represent the set of people who like contest shows.
$n(C \cup R) = n(U) - n((C \cup R)')$
$n(C \cup R) = 32 - 4$
$n(C \cup R) = 28$
$n(C \cap R) = n(C \cup R) - n(C \text{ only}) - n(R \text{ only})$
$n(C \cap R) = 28 - 13 - 9$
$n(C \cap R) = 6$
6 people like both type of shows.

16. No. e.g., The three numbers do not add up to 48. There is an overlap between sets B and C, but $B \not\subset C$. The sum of the three values in the problem is 59.
$59 - 48 = 11$
11 students must drive a car and take a bus.
$31 - 11 = 20$
20 students drive a car but do not take a bus.
$16 - 11 = 5$
5 students take a bus but do not drive a car.
There are a total of $15 + 12 = 27$ students who do not take a bus.

17. a) Sets A and B are disjoint sets.
b) Sets A and C intersect.
c) Yes; B and C; e.g., C intersecting A and A and B being disjoint says nothing about the intersection, if any, of B and C.

18. e.g., The union of two sets is more like the addition of two numbers because all the elements of each set are counted together, instead of those present in both sets.

19. a) e.g., indoor, outdoor, races
b) e.g., U = {all sports}
I = {indoor sports} = {badminton, basketball, curling, figure skating, gymnastics, hockey, indoor soccer, speed skating, table tennis, volleyball, wrestling, Arctic Sports, Dene Games}
O = {outdoor sports} = {alpine skiing, cross-country skiing, freestyle skiing, snowshoe biathlon, ski biathlon, dog mushing, snowboarding, snowshoeing, Dene Games}
R = {races} = {speed skating, alpine skiing, cross-country skiing, biathlon, dog mushing, snowboarding, snowshoeing}
c)



d) Yes. e.g., My classmate sorted the games as individual, partner and team games.

History Connection, page 35

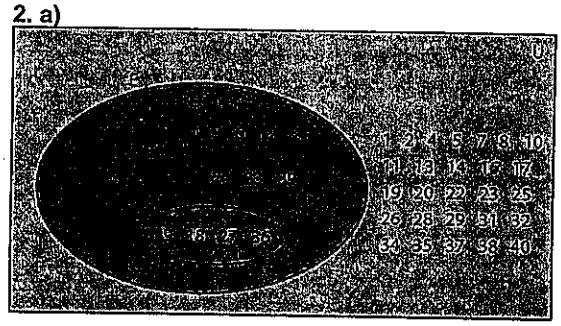
A. e.g., The "barber paradox" can be stated as follows: Suppose there is one male barber in a small town, and that every man in the town keeps himself clean-shaven. Some do so by shaving themselves and the others go to the barber. So, the barber shaves all the men who do not shave themselves. Does the barber shave himself? The question leads to a paradox: If he does not shave himself, then he must abide by the rule and shave himself. If he does shave himself, then according to the rule he will not shave himself.

B. e.g., One remarkable paradox that arises from Cantor's work on set theory is the Banach-Tarski theorem, which states that a solid, three-dimensional ball can be split into a finite number of non-overlapping pieces, which can then be put back together in a different way to yield *two* identical copies of the original ball of the same size.

Mid-Chapter Review, page 38

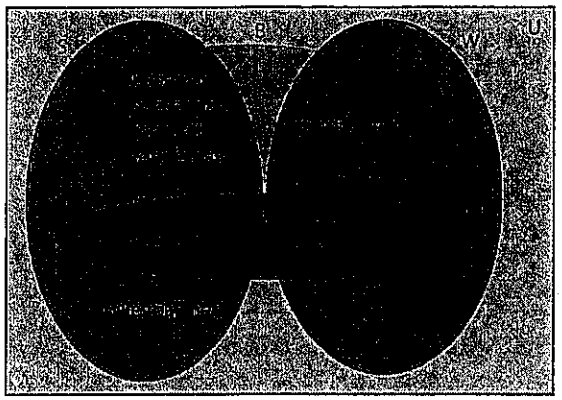
1. a) $V \subset N$, $M \subset N$, $F \subset N$, $F \subset M$
b) e.g., N = {all foods}, V = {fruits and vegetables}, M = {meats}, F = {fish}
c) No. e.g., Pasta is not part of M or V.
d) Sets V and M are disjoint, Sets V and F are disjoint.

2. a)

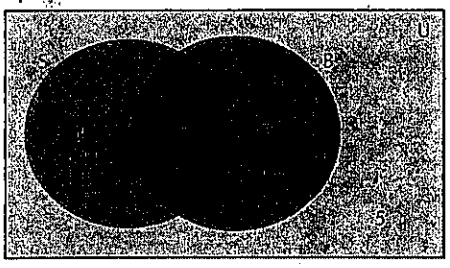


b) Sets F and S are disjoint sets.
c) i) False. e.g., 6 is in E but not F.
ii) True. e.g., All elements of S are in E.
iii) False. e.g., 9 is not a multiple of 15.
iv) True. e.g., F = {15, 30}.
v) True. e.g., A set is a subset of itself.

3. e.g., S = {summer sport equipment} = {baseball, soccer ball, football, tennis ball, baseball glove, volleyball net}
W = {winter sport equipment} = {hockey puck, skates, skis}
B = {sports balls} = {baseball, soccer ball, football, tennis ball, hockey puck}
E = {sports equipment worn on body} = {baseball glove, skates, skis}

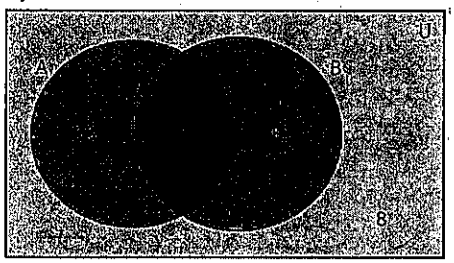


4. a) beverage or soup: 40 – 5 = 35
beverage and soup: 34 + 18 = 52
overlap: 52 – 35 = 17
17 students bought a beverage and soup.
b) only beverage: 34 – 17 = 17
only soup: 18 – 17 = 1
18 students bought only a beverage or only soup.
c)



5. a) sunglasses or hat: 20 – 5 = 15
sunglasses and hat: 13 + 6 = 19
overlap: 19 – 15 = 4
4 students are wearing sunglasses and a hat.
b) only sunglasses: 13 – 4 = 9
9 students are wearing sunglasses but not a hat.
c) only hat: 6 – 4 = 2
2 students are wearing a hat but not sunglasses.

6. a) e.g., Tanya did not put any elements in the intersection of A and B.
$n(A \cup B) = n(U) - n((A \cup B)')$
$n(A \cup B) = 40 - 8$
$n(A \cup B) = 32$
$n(A \cap B) = n(A) + n(B) - n(A \cup B)$
$n(A \cap B) = 16 + 19 - 32$
$n(A \cap B) = 3$
$n(A \setminus B) = n(A) - n(A \cap B)$
$n(A \setminus B) = 16 - 3$
$n(A \setminus B) = 13$
$n(B \setminus A) = n(B) - n(A \cap B)$
$n(B \setminus A) = 19 - 3$
$n(B \setminus A) = 16$
b)



7. Let U represent the universal set. Let D represent the set of students who have a dog. Let C represent the set of students who have a cat.
$n(C \cup D) = n(U) - n((C \cup D)')$
$n(C \cup D) = 20 - 4$
$n(C \cup D) = 16$
$n(C \cap D) = n(C) + n(D) - n(C \cup D)$
$n(C \cap D) = 8 + 8 - 16$
$n(C \cap D) = 0$
No students have a cat and a dog.

Lesson 1.4: Applications of Set Theory, page 51

1. $n(P) = p + 16$, $n(Q) = q + 21$, $n(R) = r + 18$
e.g., p Can be any number. Suppose $p = 14$. Then $n(P) = 30$.
$n(Q) = 30$, so $q = 30 - 21$ or 9
$n(R) = 30$, so r 301 – 18 or 12

2. a) $n((F \cup M) \setminus A) = 9 + 15 + 8$
$n((F \cup M) \setminus A) = 32$
b) $n((A \cup F) \setminus M) = 9 + 11 + 7$
$n((A \cup F) \setminus M) = 27$
c) $n((F \cup A) \cup (F \cup M))$
$= (9 + 11 + 7 + 9) + (15 + 8 + 4)$
$= 36 + 27$
$= 63$
d) $n(A \setminus F \setminus M) = 7$

3. e.g., Staff could look at how any David Smiths were on that bus route or they could look at the books in the bag and see how many David Smiths are taking courses that use those books.

4. P = {population surveyed}
$n(P) = 641$
L = {people wearing corrective lenses}
L' = {people not wearing corrective lenses}
$n(L') = 167$
G = {people wearing glasses}
C = {people wearing contact lenses}
$n(L) = n(P) - n(L')$
$n(L) = 641 - 167$
$n(L) = 474$
$n(G \cup C) = n(L)$
$n(G \cup C) = n(G) + n(C) - n(G \cap C)$
$474 = 442 + 83 - n(G \cap C)$
$51 = n(G \cap C)$
51 people might make use of a package deal. This is $\frac{51}{574} = 10.759...\%$ or about 10.8% of all potential customers.

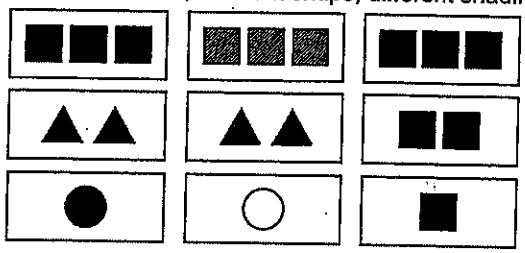
5. e.g., "Canadian Rockies," "ski accommodations," "weather forecast," "Whistler." By combining two or more of these terms, Jacques can search for the intersection of web pages related to these terms. For example, "ski accommodations" and "Canadian Rockies" is more likely to give him useful information for his trip than either of those terms on its own.

6. Using the principle of inclusion and exclusion for three sets:

$$\begin{aligned} 32 + 35 + 38 - (9 + x) - (11 + x) - (13 + x) + x &= 58 \\ 105 - 9 - x - 11 - x - 13 - x + x &= 58 \\ 72 - 2x &= 58 \\ -2x &= 58 - 72 \\ -2x &= -14 \\ x &= 7 \end{aligned}$$

7 teens are training for the upcoming triathlon.

7. • same numbers, same shapes, different shadings
• same numbers, different shape, different shading
• same numbers, different shape, different shading

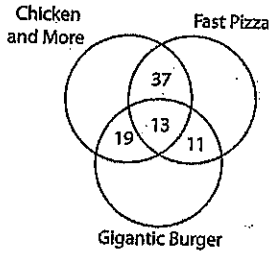


8. a) e.g., The dealer might use exterior colour, interior colour, or year.
b) e.g., The dealer might prioritize the search according to options Travis wants or by distance from where Travis lives.

9. e.g., John assumed that 90 people ate at only one restaurant for each of the 3 restaurants. He did not calculate the correct number of people eating at only one of each of the 3 restaurants.
I defined these sets.

C = {students who like only Chicken and More
F = {students who like only Fast Pizza}
G = {students who like only Gigantic Burger}

I listed the values I knew and entered them in a Venn diagram.
$n(C \cap P \setminus B) = 37$; $n(C \cap B \setminus P) = 19$;
$n(P \cap B \setminus C) = 11$
$n(C \cap B \cap P) = 13$

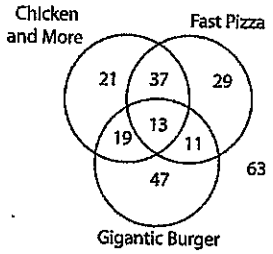


I used these figures and diagram to determine the unknown values.

$$\begin{aligned} n(C \setminus B \setminus P) &= 90 - n(C \cap P \setminus B) - n(C \cap B \setminus P) \\ &\quad - n(C \cap B \cap P) \\ &= 90 - 37 - 19 - 13 \\ &= 21 \end{aligned}$$

$$\begin{aligned} n(B \setminus P \setminus C) &= 90 - n(C \cap B \setminus P) - n(P \cap B \setminus C) \\ &\quad - n(C \cap B \cap P) \\ &= 90 - 19 - 11 - 13 \\ &= 47 \end{aligned}$$

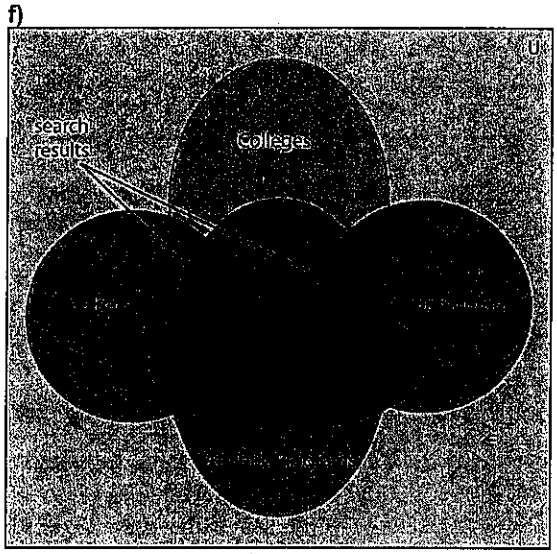
$$\begin{aligned} n(P \setminus B \setminus C) &= 90 - n(P \cap B \setminus C) - n(C \cap P \setminus B) \\ &\quad - n(C \cap B \cap P) \\ &= 90 - 11 - 37 - 13 \\ &= 29 \end{aligned}$$



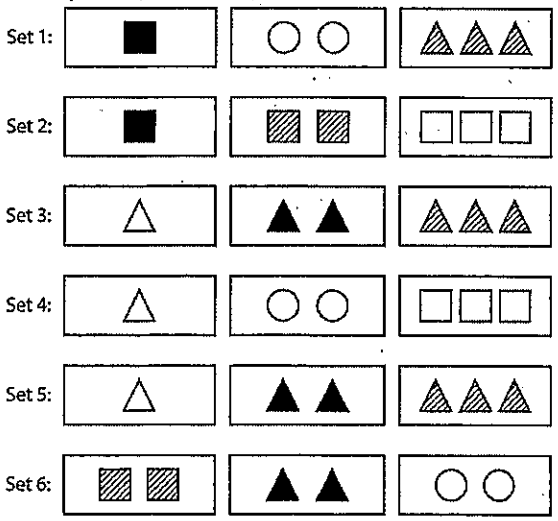
$n(R) = 21 + 29 + 47 + 37 + 19 + 11 + 13$
$n(R) = 177$

177 students like at least one of these restaurants.
$240 - 177 = 63$
So, 63 students do not like any of the restaurants.

10. a) e.g., He can search for *colleges and (Calgary or Edmonton).*
b) He should use "and" to connect the words.
c) He should use "or" to search for one or the other city.
d) e.g., *colleges and (Calgary or Edmonton) and "athletics programs" –university*
e) e.g., about 1500
f)



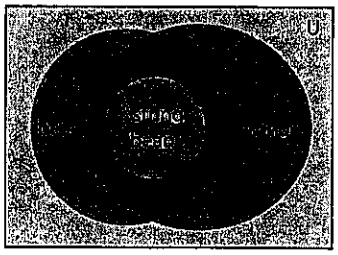
11. Set 1: different numbers, different colours, different shading, different shape
Set 2: different numbers, different colours, different shading, same shape
Set 3: different numbers, different colours, different shading, same shape
Set 4: different numbers, same colour, same shading, different shape
Set 5: different numbers, same colour, different shading, same shape
Set 6: same number, different colours, different shading, different shape



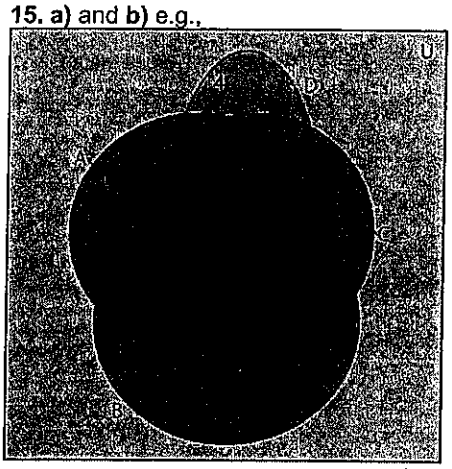
12. a) $n(D)$, the total number of cards in the deck: there are 3 shapes, 3 colours, 3 numbers, and 3 shadings, so in total there are $3 \cdot 3 \cdot 3 \cdot 3$ or 81 cards.
b) $n(T)$, the total number of triangle cards in the deck: there are 3 colours, 3 numbers, and 3 shadings, so in total there are $3 \cdot 3 \cdot 3$ or 27 triangle cards.
c) $n(G)$, the total number of green cards in the deck: 3 shapes, 3 numbers, and 3 shadings, so in total, there are $3 \cdot 3 \cdot 3$ or 27 green cards.
d) $n(S)$, the total number of cards with shading: there are 27 cards with striped shading and 27 cards with solid shading, so there are 27 + 27 or 54 cards with shading.
e) $n(T \cup G)$: there are 27 triangle cards and 27 green cards, but 9 triangle cards are also green, so there are 54 – 9 or 45 cards that have triangles or are green.
f) $n(G \cap S)$: there are 27 green cards. Since 2/3 of the cards have either striped shading or solid shading, 18 cards are both green and have shading.

13. a) 36 sites would appear in a search for fishing boats. There are 35 sites that involve boats, 20 of which deal with fishing boats. 21 sites involve fishing, but these sites include the 20 sites that deal with fishing boats.
b) e.g., Because *fishing* and *boats* will turn up sites that deal with boats and fishing, but not just fishing boats.
c) 20 of the 21 fishing sites deal with fishing boats, so 1 site would not have boats.

14. e.g., No, they did not get the same results. Elinor got all of James' results, plus others dealing with either string or bean, but not both.



15. a) and **b)** e.g.,



c) e.g., $1 = A \setminus (B \cup C \cup D)$
$2 = B \setminus (A \cup C \cup D)$
$3 = C \setminus (A \cup B \cup D)$
$4 = D \setminus (A \cup B \cup C)$
$5 = (A \cap B) \setminus (C \cup D)$
$6 = (A \cap C) \setminus (B \cup D)$
$7 = (A \cap D) \setminus (B \cup C)$
$8 = (B \cap C) \setminus (A \cup D)$
$9 = (B \cap D) \setminus (A \cup C)$
$10 = (C \cap D) \setminus (A \cup B)$
$11 = (A \cap B \cap C) \setminus D$
$12 = (A \cap B \cap D) \setminus C$
$13 = (A \cap C \cap D) \setminus B$
$14 = (B \cap C \cap D) \setminus A$
$15 = A \cap B \cap C \cap D$

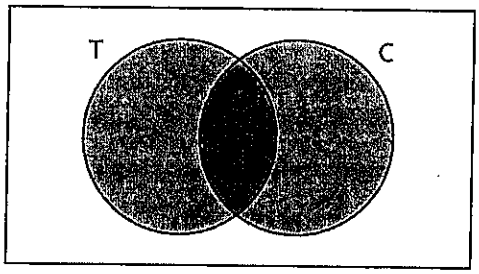
16. e.g., Let B = {blue}, Y = {yellow}, R = {red}, and G = {green}. There is no area representing $(B \cap R) \setminus (G \cup Y)$ or $(G \cap T) \setminus (B \cup R)$.

Math in Action: Relevant Hits, page 54

e.g.,
• I decided to research texting in relation to driving safely.

Search Words	Number of Hits
texting and driving	3 830 000
texting while driving	976 000
"texting while driving"	599 000
"texting while driving in Canada"	8

• I had way too many hits for texting and driving. I figured out that the issue is texting while driving, so I tried that combination. Putting quotes around it netted even fewer results. Since I live in Canada, I was interested in what's happening here, so I added Canada to my search. Then I tried "texting while driving in Canada". That really cut down the hits to a manageable number.



Let T represent "texting while driving" sites, and C represent Canada sites. The overlap of the two circles represents the sites that contain both "texting while driving" and Canada.
• The search engine's Advanced Search feature allows you to exclude any sites that contain certain words from your search.

Applying Problem-Solving Strategies: Analyzing a Logic Puzzle, page 55

C. e.g., In this solution, squares are numbered from 1 to 9, from the top left to the bottom right, as on the numeric pad of a telephone. First, I examined the coloured squares. I know from the first clue that either square 5 or 6 must be blue. The second clue tells me that either square 6 or 9 is blue. The fifth clue tells me that either square 7 or 8 is blue. So I know that either 7 or 8 must be blue, and one or two of 5, 6, and 9 must be blue.

Also, either square 4 or 5 is red, either square 6 or 9 is red, and square 2 or 5 is yellow.
Since squares 6 and 9 are red or blue, they cannot be yellow.
I know that there is a heart in either square 1 or 2, and two hearts in either 2 and 4, or 3 and 5. The yellow heart is in either 2 or 3. All of the hearts are in the first two rows.
I decided to begin to place the colours and symbols, knowing that I might need to move them around. I put hearts in squares 1, 3, and 5.
Since the three hearts must be three different colours, I think that square 5 will be a blue heart, which makes square 1 a red heart, because the blue colours appear to be in the second and third rows.

red heart		yellow heart
	blue heart	red or blue
		red or blue

I know that either square 4 or 5 must be red. Since there is a blue heart in square 5, square 4 must be red, which makes square 7 blue, according to the fifth clue.
The sixth clue indicates that either square 1 or 4 is a star. Since I have a heart in square 1, this means that a star must be in square 4; so, it is a red star.
I now have three red squares and three blue squares, so the remaining two squares, square 2 and 7, must be yellow.

red heart	yellow	yellow heart
red star	blue heart	red or blue
blue	yellow	red or blue

The fourth clue tells me that either 3 or 6 is a pentagon. Since I already have a heart in square 3, square 6 must contain the pentagon. This clue also tells me that a star must be in square 8, and since I know square 8 is yellow, it is a yellow star.

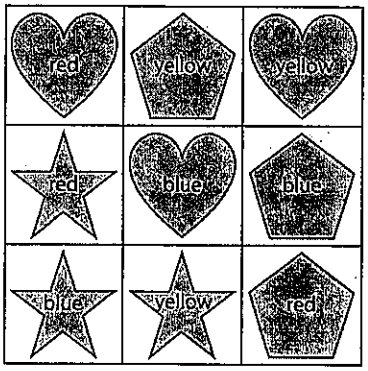
Two yellow shapes have been placed. The yellow pentagon belongs in square 2.

red heart	yellow pentagon	yellow heart
red star	blue heart	red or blue pentagon
blue	yellow star	red or blue

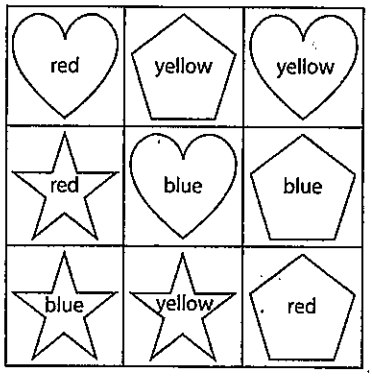
I used the sixth clue to determine whether square 9 is red or blue. The star is in square 4, so square 9 must be red. The only missing red shape is the pentagon, so it is a red pentagon, and square 6 is the blue pentagon. Finally, the blue star belongs in square 7.

red heart	yellow pentagon	yellow heart
red star	blue heart	blue pentagon
blue star	yellow star	red pentagon

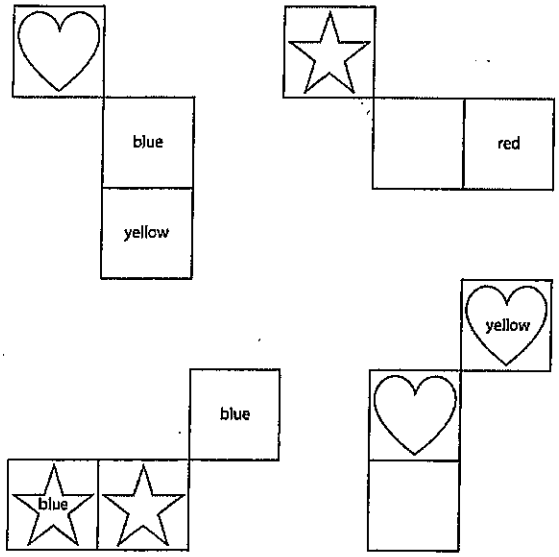
I double-checked my clues. My puzzle is correct.
Solution:



D. The solution is:



My clues are:



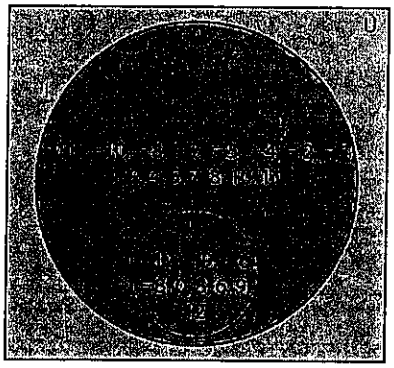
F. To make the puzzle easier, I could give more clues where both the colour and shape are given. Or, I could give the shapes in a diagonal, or a group of colours or shapes that would show one square in every row and column.
To make the puzzle harder, I could not give any clues with both the colour and the shape, or I could make the pieces smaller or without angles so there are more possibilities for their location in the 3 by 3 grid.

Chapter Self-Test, page 56

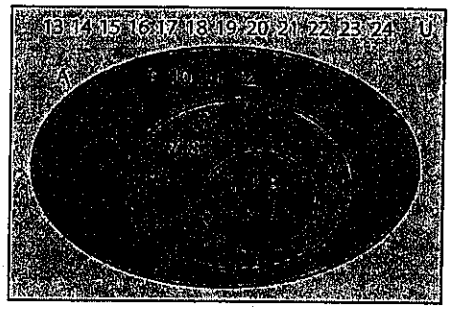
1. a) M = {Eric Ashdown, Art Napoleon, Don Amero, Lucie Idlout}
b) S = { Terry-Lynn Williams Davidson, Art Napoleon, Don Amero, Lucie Idlout}
c) $M \cap S$ = { Art Napoleon, Don Amero, Lucie Idlout}
d) $M \setminus S$ = {Eric Ashdown}

2. a) $A = \{-12, -9, -6, -3, 0, 3, 6, 9, 12\}$
$B = \{x \mid -12 \leq x \leq 12, x \in I\}$
$A \cup B = \{x \mid -12 \leq x \leq 12, x \in I\}$
$= \{-12, -11, -10, -9, -8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$
$n(A \cup B) = 25$
$A \cap B = \{-12, -9, -6, -3, 0, 3, 6, 9, 12\}$
$n(A \cap B) = 9$

b) Draw a Venn diagram of these two sets.



3. a)



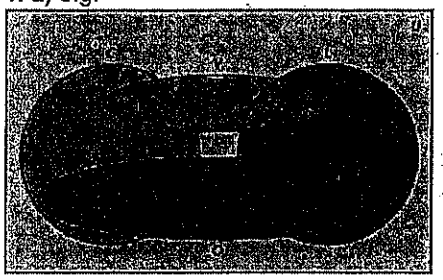
Set C is inside set A, therefore $C \subset A$.
b) A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}
B = {1, 2, 3, 4, 5, 6, 7, 8}
$A \cup B$ = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}
$n(A \cup B) = 12$
C = {1, 2, 3, 4, 5, 6}
$A \cap C$ = {1, 2, 3, 4, 5, 6}
$n(A \cap C) = 6$
c) $A \cap B$ = {1, 2, 3, 4, 5, 6, 7, 8}
$A \cap B \setminus C$ = {7, 8}
d) $A \cup B \cup C$ = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}
$(A \cup B \cup C)'$ is all the elements not in $A \cup B \cup C$.
$(A \cup B \cup C)'$ = {13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24}

4. Let U represent the universal set. Let W represent the students who drink bottled water. Let L represent the students who follow a low fat diet. Let F represent the students who eat fruit. We know 15% of students do all three, so that number is the three-way intersection.
We know 22% drink bottled water and follow a low-fat diet, so $n(L \cup W / F) = 22 - 15 = 7$.
Similarly, $n(W \cup F / L) = 27 - 15 = 12$ and
$n(L \cup F / W) = 23 - 15 = 8$.
From here, I know 50% of the students drink bottled water, 56% eat fruit, and 43% follow a low-fat diet.
$W / L \cup F = 50 - 15 - 7 - 12 = 16$
$F / W \cup L = 56 - 15 - 12 - 8 = 21$
$L / W \cup F = 43 - 15 - 7 - 8 = 13$

To determine the percent of students who do not drink bottled water, eat fruit or follow a low-fat diet we need $(W \cup F \cup L)'$.
$(W \cup F \cup L) = 16 + 13 + 21 + 12 + 7 + 8 + 15 = 92$
$(W \cup F \cup L)' = 100 - 92 = 8$
Therefore, 8% of students do not drink bottled water, eat fruit or follow a low-fat diet.

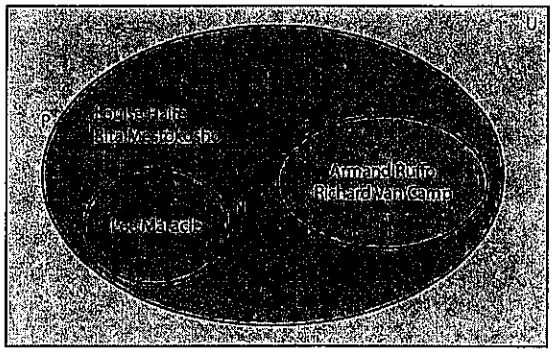
Chapter Review, page 58

1. a) e.g.



b) $C' = L$, $L' = C$, $O' = V$, $V' = O$
c) C and L, O and V
d) No, there are no letters that fit in two categories.

2. Let U represent the universal set of writers. Let P represent the set of poets. Let N represent the set of novelists, and let F represent the set of fiction writers.
Subset fiction writer, F = {Armand Ruffo, Richard Van Camp}



3. People who do not use a landline: 96 – 68 = 28
People who use both a cellphone and a landline:
84 – 28 = 56
56 people use both a cellphone and a landline.

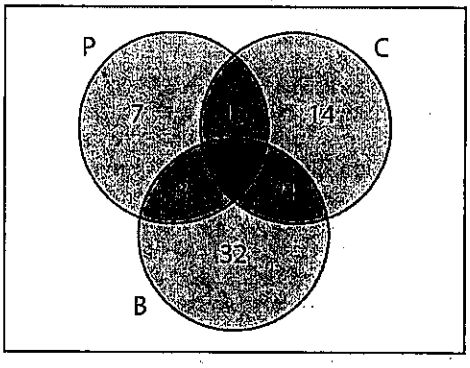
4.a) black hair or blue eyes: 28 – 9 = 19
Since 19 students have black hair, all students with blue eyes have black hair.
Of the 19 students with black hair, 8 have blue eyes.

b) only black hair: 19 – 8 = 11
Of the 19 students with black hair, 11 students have black hair but not blue eyes.
c) only blue eyes: 8 – 8 = 0
No students have blue eyes but not black hair.

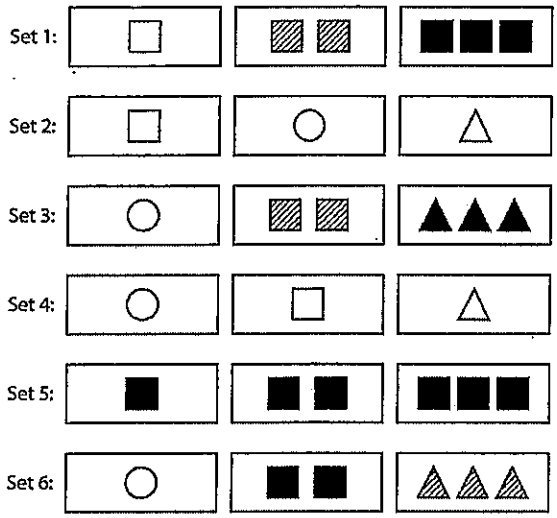
5. Teenagers who only want to go canoeing:
28 – 20 = 8
Teenagers who only want to go swimming:
45 – 20 = 25
Total number of teenagers: 8 + 25 + 20 + 11 = 64
There are 64 teenagers at the camp.

6. Number of people asked: 40
Number who like romance novels; 10
Number who like horror novels: 13
Number who do not like either: 18
Romance or horror or both: 40 – 18 = 22
Both romance and horror: 10 + 13 – 22 = 1
One person likes both romance and horror novels.

7. Students taking 3 sciences: 12
Students taking 2 sciences:
Physics and Chemistry: 27 – 12 = 15
Physics and Biology: 15 – 12 = 3
Chemistry and Biology: 33 – 12 = 21
Students taking 1 science:
Physics: 37 – 15 – 3 – 12 = 7
Chemistry: 62 – 15 –21 – 12 = 14
Biology: 68 – 3 – 21 – 12 = 32
Total Number of Students:
12 + 15 + 3 +21 + 7 +14 + 32 = 104
There were 104 students in Grade 12 that year.



8. Set 1: Different numbers, same shape, different colours.
Set 2: Same number, different shape, different colour.
Set 3: Different numbers, different shape, different colour.
Set 4: Same number, different shape, different colour.
Set 5: Different number, same shape, different colour.
Set 6: Different number, different shape, different colour.



Chapter Task, page 59

A. I would organize most of the animals according to geographic region. I would use three sets: America (**A**), Africa (**F**), and Asia/Australia (**K**). I would need to consider both indoor and outdoor animals, as well as animals that need room to roam or graze. I could put birds, insects, and reptiles as subsets of each geographic region, or I could have a separate building for them and categorize them inside this building. If I have fish, it would make sense to put them all in an aquarium building, rather than have several buildings. I would also need water for birds who swim. I would need to have enclosures for small animals and for animals that require controlled climate conditions. I could put predators and prey in the same areas, but not in the same compounds.

B. and **C.** Set America (**A**) will have three subsets: North America (**N**), South America (**S**), and Central America (**C**). Central America will intersect sets North America and South America. Set Africa (**F**) will have two subsets: Savanna (**V**) and Rainforest (**R**). Some of the animals from sets Africa and America will need roaming and grazing room, so set Graze (**G**) will intersect both Africa and America. The bears, lions, and wolves will need large areas to roam and should be kept apart from the other animals, so they will have a separate area (**L**). I am also going to separate the Australia/Asia (**K**) set of animals as a special attraction area. This will have two subsets: indoor (**D**) and outdoor (**T**).

I decided to put the reptiles and insects with the indoor animals from Australia (**H**), since I thought they would thrive best there. I decided to put the birds in a separate area. I could not show the intersection of the sets on my diagram, but I listed the birds. Also, I could not show the intersection of the jaguar from South America with the area for the bears, lions, and wolves, but I put it in the top area. My sets will contain the following animals:
America:
N = { moose, cougar, lynx, grizzly bear, polar bear, bison, elk, raccoon, lynx, Arctic fox, Arctic wolf, snowy owl, beaver, ferret, prairie dog, flamingo, swan}
S = { tarantula, black widow spider, blue poison dart frog, boa constrictor, two-toed sloth, tamarin, marmoset, jaguar, spider monkey, macaw, llama}
C = {boa constrictor, poison dart frog, ocelot, jaguar, spider monkey}
I = { ferret, black widow spider, prairie dog, burrowing owl, beaver, boa constrictor, blue poison dart frog, marmoset, tamarin, tarantula, two-toed sloth, macaw}
O = { moose, cougar, lynx, grizzly bear, polar bear, bison, elk, raccoon, lynx, Arctic fox, Arctic wolf, snowy owl, jaguar, spider monkey, llama}

Africa:
R = { tortoise, mandrill, pygmy hippopotamus, fruit bat, gorilla}
V = { meerkat, elephant, zebra, lion, cheetah, crane, stork, baboon, hippopotamus, ostrich, hyena}
G = { moose, bison, elk, llama, elephant, zebra, ostrich}

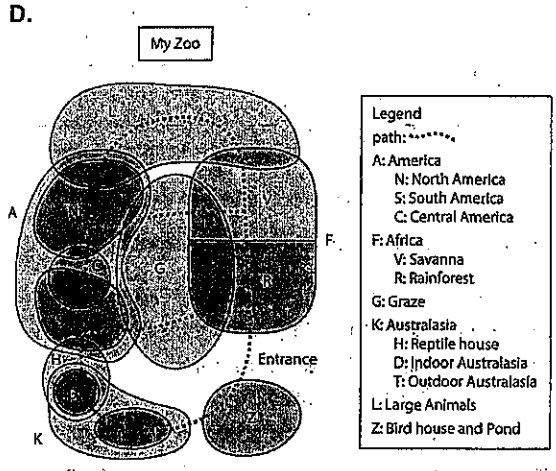
Australia:
D = { tree boa, frilled lizard, komodo dragon, bearded dragon, tree python, kookaburra, tree kangaroo, sugar glider}
T = {kangaroo, wombat}

Reptile House
H = { tarantula, blue poison dart frog, boa constrictor, black widow spider, tree boa, frilled lizard, komodo dragon, bearded dragon, tree python, tree kangaroo, wombat, sugar glider}

D is now a subset of **H**.

Birds
Z = {snowy owl, flamingo, macaw, crane, stork, ostrich, swan}

D.



E Yes, my zoo is easy to navigate. There is a wide-open space at the entrance, with the feature birds and pond as you go in. The animals are categorized, so you can just walk around to see the different continents.
The more dangerous animals are located together, and there is a common area for the roaming and grazing animals. I think visitors will find my zoo easy to navigate. They will be able to describe it to their friends and recommend it. Attendance will increase, because their friends will want to check it out for themselves.
Therefore, the conditional statement is true for my zoo.
To form the inverse of a conditional statement, I need to negate the hypothesis and the conclusion. So, the inverse of the conditional statement is this: If visitors do not find the zoo easy to navigate, then they are not more likely to recommend it to their friends, and attendance will not increase. The inverse is true. If visitors find the zoo confusing and awkward to navigate, they may not return and they will not recommend the zoo to their friends. As a result, the attendance will not increase, and perhaps it will even decline.

Chapter 1 Diagnostic Test, TR page 54

1. A die has six possible outcomes: 1, 2, 3, 4, 5, or 6. Tossing a coin has two possible outcomes: heads (H) or tails (T).

	1	2	3	4	5	6
H	H, 1	H, 2	H, 3	H, 4	H, 5	H, 6
T	T, 1	T, 2	T, 3	T, 4	T, 5	T, 6

2. Answers will vary. e.g., long-necked birds = {flamingo, ostrich, stork, swan}
short-necked birds = {bluebird, duck, robin, sparrow}

3. e. g., shapes with curved sides:

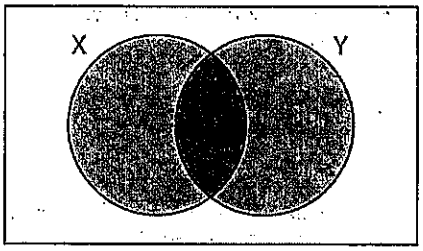
shapes with straight sides:

4. e. g., the, pen, red, son, odd, boo, nut, cat, dog, too, bib, tam, did, fog, mat
S = {words with two of the same letters}
S = {odd, boo, too, bib, did}
V = {words with the letter O}
V = {son, odd, boo, dog, too, fog}
The words belonging in both groups are: odd, boo, too.
The words belonging in neither group are: the, pen, red, nut, cat, dog, tam, mat.

5. a) $R = \{x \mid 1 \leq x \leq 7, x \in \mathrm{N}\}$
$R = \{1, 2, 3, 4, 5, 6, 7\}$
b) $T = \{3x \mid -3 \leq x \leq 2, x \in \mathrm{I}\}$
$T = \{-9, -6, -3, 0, 3, 6\}$

Review of Terms and Connections, TR page 56

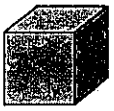
1. a) ii) Venn diagram



b) v) outcome table

		Nickel	
		H	T
Dime	H	(H, H)	(H, T)
	T	(T, H)	(T, T)

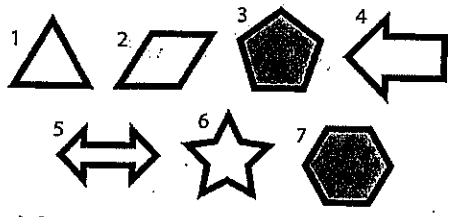
c) iv) set-builder notation;
$Y = \{x \mid 3 < x < 10, x \in \mathrm{N}\}$
d) iii) attributes; 3-D, cube, square faces



object, cube, square faces

e) i) set notation; $A = \{2, 4, 6, 8, 10\}$

2. e. g.,



a) Sort according to shading: Let H represent the set of hollow shapes and S represent the set of solid shapes.
$H = \{1, 2, 4, 6\}$, $S = \{3, 7\}$
b) Sort according to number of sides: Let O represent the set of shapes with an odd number of sides and E represent the set of shapes with an even number of sides.
$O = \{1, 3, 4\}$, $E = \{2, 5, 6, 7\}$

3. a) 6.4 is a decimal, so it is in the rational number system, Q and the real number system, R.
b) $\sqrt{36}$ is a square root number. In this case, the number is 6 which is a natural, whole, rational, real integer. It belongs to N, W, I, Q and R.
c) –123 is a negative number so it belongs to the integer number system, I. It also belongs to the rational number system, Q, which includes all integers as well as the real number system, R.
d) –8.5 is a decimal, so it is in the rational number system, Q and the real number system, R.
e) $7^{\frac{1}{2}}$ is another way of writing a square root, so the number belongs to the irrational number system, $\overline{Q}$ and the real number system, R.

4. $T = \{5, 6, 7, \ldots, 97, 98, 99\}$
Let a represent the numbers in the set T.
$T = \{a \mid 5 \leq a \leq 99, a \in I\}$

5. The outcomes of a six-sided die are: 1, 2, 3, 4, 5, or 6. The outcomes of a four-sided die are 1, 2, 3, or 4.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10

6. e.g., **a)** odd = {3, 9, 15, 21, 27, 33}
even = {6, 12, 18, 24, 30, 36}
b) Yes, there is more than one solution. For example:
numbers whose digits add to 9 = {9, 18, 27, 36}
numbers whose digits do not add to 9 =
{3, 6, 12, 15, 21, 24, 30, 33}

7. a) –789 is a negative number so it belongs to the integer number system, I. It also belongs to the rational number system, Q, which includes all integers. It also belongs to the real numbers, R.
b) 62.3 is a decimal, so it is in the rational number system, Q. It also belongs to the real numbers, R.
c) $-98\frac{3}{4}$ is a decimal, so it is in the rational number system, Q. It also belongs to the real numbers, R.
d) 2.349 583 430 723 423 445 429 743... is a non-repeating, non-terminating decimal number, so it belongs to the irrational number system, $\overline{Q}$. It also belongs to the real numbers, R.
e) $\sqrt{59}$ is a square root number, so it belongs to the irrational number system, $\overline{Q}$. It also belongs to the real numbers, R.
f) cos 116° = 0.971..., which is a non-repeating, non-terminating decimal number, so it belongs to the irrational number system, $\overline{Q}$. It also belongs to the real numbers, R.
g) 19 387 is a whole, natural, rational integer, and a real number, so it belongs to N, W, I, Q, and R.
h) tan 45° = 1, which is a natural number, so it I belongs to the natural number system, N. It is also in the whole (W), integer (I), and rational (Q) number systems, since all of these systems include the natural numbers.

8. a) $K = \{a \mid -3 \leq a \leq 5, a \in I\}$
$K = \{-3, -2, -1, 0, 1, 2, 3, 4, 5\}$
b) $M = \{2p \mid 1 \leq p \leq 4, p \in N\}$
$M = \{2, 4, 6, 8\}$
9. a) $Z = \{x \mid x \geq 100, x \in N\}$
Z = {all natural numbers 100 or greater}

b) $L = \{x \mid x = \frac{y}{4}, 1 \leq x \leq 10, x \in N\}$

The range of x is the natural numbers from 1 to 10 inclusive. So, the values of y must be natural numbers that are multiples of 4 that range from 4 to 40 inclusive.

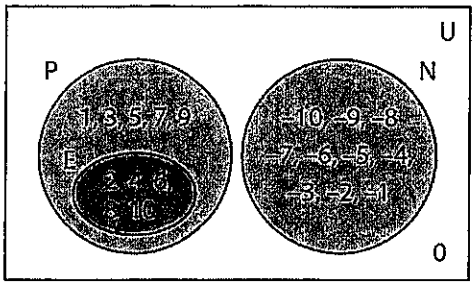
For $x = 1$, $1 = \frac{y}{4}$, so $y = 4$, and so on.

L = {all multiples of 4 from 4 to 40}

10. a) W = {integers from –25 to 250}
$W = \{x \mid -25 \leq x \leq 250, x \in I\}$
b) E = {even positive numbers greater than 8}
$E = \{2x \mid x \geq 5, x \in N\}$

Chapter 1 Test, TR page 62

1. a) and **b)**



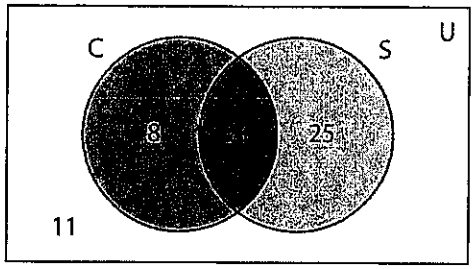
c) Sets P and N are disjoint sets. Sets E and N are disjoint sets.
d) Yes, Set E is a subset of set P, because set P contains all the elements of set E.
e) No, set P' does not equal set N because 0 does not belong to set P or set N. Set P' includes all the negative numbers in set N, plus 0.

2. a) $n(A) = 7$
b) $n(B) = 2$
c) $n(A \cap B) = 1$
d) $n((A \cup B)') = 7$
e) $n(U) = 15$

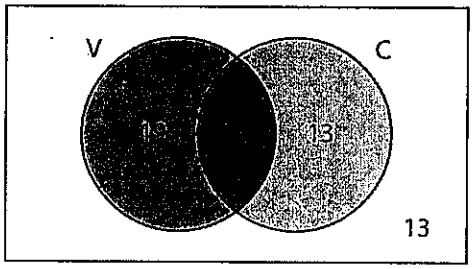
3. $A = \{x \mid x \leq 12, x \text{ is a prime number}\}$
$A = \{2, 3, 5, 7, 11\}$
$B = \{x \mid 1 < x \leq 10, x \text{ is an even number}\}$
$B = \{2, 4, 6, 8, 10\}$
$n(A) = 5$
$n(B) = 5$
$n(A \cap B) = 1$
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$n(A \cup B) = 5 + 5 - 1$
$n(A \cup B) = 9$

4. Let C represent using a cellphone, and L represent using a land line.
$n(C) + n(L) = 152$
This is 56 more than the number of people surveyed, so 56 people used both.

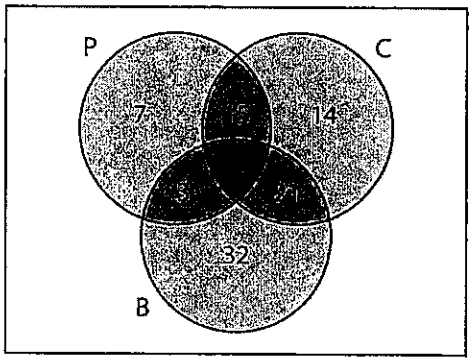
5. I drew a Venn diagram, and wrote 20 where the sets for canoeing and swimming overlap. I wrote 11 outside those sets, for those do not like either sport. Since 28 campers want to canoe, then there are 28 – 20 or 8 campers who only want to canoe. Since 45 campers want to swim, then there are 45 – 20 or 25 campers who only want to swim. I added the numbers in each region. There are 11 + 8 + 20 + 25 or 64 campers in all.



6. 60 – 13 = 47 people had ice cream or chocolate sauce.
Of these:
47 – 34 = 13 did not have vanilla ice cream.
47 – 28 = 19 did not have chocolate sauce.
47 – 13 – 19 = 15 had vanilla ice cream and chocolate sauce.



7. 12 students took all three sciences, and 27 students took physics and chemistry, so
27 – 12 = 15 students took physics and chemistry but did not take biology. Similarly, 15 students took physics and biology, so 15 – 12 = 3 took physics and biology, but did not take chemistry. And, 33 students took chemistry and biology, so 33 – 12 = 21 students took chemistry and biology, but did not take physics. That means:
37 – 15 – 12 – 3 = 7 students took only physics.
62 – 15 – 12 – 21 = 14 students took only chemistry.
68 – 3 – 12 – 21 = 32 students took only biology.



There were 104 grade 12 students.